LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc., DEGREE EXAMINATION – **STATISTICS**

FOURTH SEMESTER – **NOVEMBER 2013**

ST 4502/ST 4501 – DISTRIBUTION THEORY

Date : 08/11/2013 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

PART – A (10 x 2 = 20)

Answer ALL questions:

1. A deck of cards is well shuffled. Find the probability of getting an ace.
2. If *f(x)* = $\begin{cases} 2x\ ,\ 0 < x < 1 \\ 0\ ,\ \ otherwise \end{cases}$ be the probability density function of X, find the mean of X.
3. Find the first two cumulants of Poisson distribution.
4. Find the moment generating function of Negative binomial distribution.
5. Write the probability function of a multinomial distribution.
6. If X has Normal distribution with mean 0 and variance 1, find the distribution of X^2.
7. Write the m.g.f. of Gamma distribution with two parameters.
8. If $X \sim \beta_1(a,b)$ then prove that $1 - X \sim \beta_1(b,a)$
9. State the relation between *t* and *F* distributions.
10. Write the p.d.f. of the first order statistic.

PART – B (5 X 8 = 40)

Answer any FIVE questions:

11. Two cards are drawn at random (without replacement) from a standard 52 card deck. Let X be the number of aces that occur and Y be the number of spades that occur. Derive p(x,y) and compute P(X>Y).
12. Define geometric distribution and find its mean and variance.
13. Derive the p.d.f. of the k^{th} order statistic.
14. Show that the exponential distribution has lack of memory property.
15. Prove that Laplace distribution is symmetric distribution. (P.T.O)

16. If X and Y are two independent Gamma varaiates, find the distribution of X/(X+Y) and X+Y.

17. State and prove Lindeberg Levy central limit theorem.

18. Prove that for a normal distribution $\mu_{2n+1} = 0 \text{ and } \mu_{2n} = \sigma^{2n}(2n-1)(2n-3)\ldots3.1$

PART – C (2 X 20 = 40)

Answer any TWO questions:

19. (a) Two random variables X and Y have the joint density

$$f(x,y) = \begin{cases} 2-x-y\ , & 0<x<1,\ 0<y<1 \\ 0\ , & otherwise \end{cases}$$

Find the marginal and conditional density functions of X and Y and find E(X | Y) and V(X | Y)

(b) Find the moment generating function of Binomial distribution and hence find mean and variance.

20. (a) Define F distribution and derive its probability density function.

(b) Obtain the marginal distributions and the conditional distributions in the case of Bivariate normal distribution.

21. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$

(a) Show that the sample mean and sample variance are independent.

(b) Obtain the distribution of the sample mean and sample variance.

22. (a) State and prove the linearity property of Normal distribution.

(b) Define Beta distribution of first kind and find its mean and variance.

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